Analysis of Radial Constraint Representation Methods for Distribution Networks

Chengsi Xu, *Shufeng Dong
College of Electrical Engineering
Zhejiang University
Hangzhou, China
dongshufeng@zju.edu.cn

Yuquan Liu, Shunqi Zeng
Guangzhou Power Supply Co., Ltd.
Guangzhou, China
bdlyq007@163.com

Shupeng Zhang, Yifan Xu
College of Electrical Engineering
Zhejiang University
Hangzhou, China

***Abstract*—In distribution network optimization problems, radial topology is an important constraint to be considered. The review of existing radial constraint representation methods (RCRMs) has not been reported yet, while appropriate method should be selected according to specific network to improve solving efficiency. This paper is devoted to a new RCRM and a summary of the existing RCRMs. Firstly, based on the idea of loop disconnection, a sufficient and necessary condition for distribution network topology to be radial is proposed, and an algorithm for searching all loops in a network is introduced. Then, the differences and characteristics of various RCRMs are analyzed. Finally, the introduced RCRMs are applied to the problem of service restoration reconfiguration. Simulation result on an actual distribution network in Fuzhou, China compares the computing performance of different methods.**

***Index Terms*—radial constraint, distribution network, loop disconnection, service restoration reconfiguration**

I. Introduction

Distribution networks typically operate with a radial topology, which is a difficult point to be considered in problems such as network reconstruction and planning [1]. Many researchers have studied the constraints that a radial distribution network should satisfy, and have put forward a variety of mathematical models.

Some researchers have introduced a spanning tree model to describe the radial structure of the distribution network [2-3], which guarantees that all nodes in a network except substation nodes have one and only one parent node. Others use power flow balance constraints to ensure connectivity of the distribution network [4-5], but in fact, the power flow constraints is not a sufficient condition for network connectivity, which limits the universality of the model. This problem can be solved by injecting a small value of power in all non-substation nodes [6], but it will introduce error into calculation results. Furthermore, a few researchers propose that the radial constraint of the distribution network can be expressed as follows: For any load node in a network, only one of its power supply paths is connected [7]. Recently a virtual demand model is proposed [8]. In this model, all nodes except substation nodes have one unit of virtual demand which only can be provided by substation nodes, so as to establish virtual power flow constraints to ensure network connectivity.

In distribution network optimization problems, it is of practical significance to find a simpler radial constraint representation method (RCRM) to reduce the complexity of the mathematical model and improve solving efficiency. In addition, different RCRM have different characteristics. For networks with different topological structure characteristics, the complexity of the model established by the same method is also different. Therefore, it is necessary to analyze and summarize the advantages and disadvantages of different methods and their application scenarios.

The rest of this paper is organized as follows: Section II introduces the existing RCRMs for distribution networks and their mathematical models. Section III proposes a RCRM based on loop disconnection and an algorithm for searching all loops in a distribution network. Section IV compares and analyzes the characteristics of different RCRMs and their application scenarios. Section V applies the introduced RCRMs to the service restoration reconfiguration problem of the distribution network. In Section VI, a specific case is tested to compare the computing performance of different RCRMs. Conclusions are drawn in Section VII.

II. Existing RCRMs for Distribution Networks

The following two conditions are necessary and sufficient to ensure that the distribution network topology is radial, and quite a few researchers have proposed RCRMs based on these two conditions [2-6], [8].

- Condition 1: There are N-N_s closed branches in the distribution network.
- Condition 2: The distribution network is connected.

Where N is the total number of nodes in the network; N_s is the number of substation nodes in the network.

The formula for condition 1 is as follows:

$$\sum_{b=1}^{B} x_b = N - N_s \tag{1}$$

Where B is the total number of branches in the distribution network; b is the branch number; x_b is the state of the bth branch, whose value is 1 when the bth branch is closed and 0 when the bth branch is disconnected.

Condition 2 involves network connectivity, which is a property related to the whole network. Different methods have

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been proposed to model this condition in literature. The spanning-tree model [2-3] introduces two variables for each branch to indicate whether the nodes at both ends of the branch are the parent nodes of each other, that is, the hierarchical relationship of nodes. Thus, condition 2 can be represented by equation 2-4.

$$\sum_{n_j \in \Gamma_i} \alpha_{ij} = 0,\ n_i \in \Omega_{\mathrm{s}} \tag{2}$$

$$\sum_{n_j \in \Gamma_i} \alpha_{ij} = 1,\ n_i \in \Omega_{\mathrm{u}} \tag{3}$$

$$\alpha_{ij} + \alpha_{ji} = x_b \tag{4}$$

Where n_i and n_j are the nodes at both ends of the bth branch; α_{ij} is the introduced binary variable, whose value is 1 when n_j is the parent node of n_i and 0 when n_i is the parent node of n_j; Ω_{s} is the set of substation nodes; Ω_{u} is the set of load nodes; Γ_i is the set of adjacent nodes of n_i.

In the virtual demand model [8], it is assumed that each load node has one unit of virtual demand, and only substation nodes can provide virtual demand. Based on this, the virtual power flow constraints are established, and condition 2 is represented by equation 5-7.

$$v_{ij} = -v_{ji} \tag{5}$$

$$\sum_{n_j \in \Gamma_i} v_{ji} = 1,\ n_i \in \Omega_{\mathrm{u}} \tag{6}$$

$$-x_b N \le v_{ij} \le x_b N \tag{7}$$

Where v_{ij} is the virtual flow from n_i to n_j.

In the path-based model [7], radial constraint is expressed equivalently with the following two conditions:

- For any load node in the network, one and only one path in its power supply path set is connected.
- If a power supply path is connected, any power supply path included in it should also be connected.

According to the above conditions, the radial constraint of the network can be represented by equation 8-9.

$$\sum_{\zeta_p \in \Pi_i} W_p = 1,\ \ p = 1, 2, \cdots, P \tag{8}$$

$$W_p \le W_q \tag{9}$$

Where p is the number of power supply path; P is the total number of power supply paths in the network; W_p is the state of the pth power supply path, whose value is 1 when the pth power supply path is connected and 0 when the pth power supply path is disconnected; ζ_p is the pth power supply path in the network; Π_i is the power supply path set of n_i; W_q is the state of the qth power supply path, which satisfies $W_q \subset W_p$.

III. Radial Constraint Representation Method Based on Loop Disconnection

A. Mathematical Model

This section proposes a new RCRM based on loop disconnection. Different from the existing methods, this paper takes condition 3 as one of the conditions that radial topology should satisfy, and proposes proposition 1, as follows:

- Condition 3: All loops in the network are disconnected.
- Proposition 1: condition 1 and condition 3 are necessary and sufficient conditions to ensure distribution network topology to be radial.

The formula for condition 1 is as follows:

$$\sum_{m=1}^{M_l} x_{lm} \le M_l - 1,\ \ l = 1, 2, \cdots, L \tag{10}$$

Where L is the number of loops in the distribution network; M_l is the number of branches in the lth loop; x_{lm} is the state of the mth branch in the lth loop.

The proof of proposition 1 is as follows:

Obviously, the following equation is true.

$$\sum_{c=1}^{C} Q_c = N \tag{11}$$

Where C is the number of connected subgraphs in the network; Q_c is the number of nodes in the cth connected subgraph.

Proof of sufficiency: Condition 3 ensures that all loops in the network are disconnected and there is at most one substation node in each connected subgraph. Thus, the network is not radial only if there are isolated islands without substation.

Assume that the network is not radial, then as described above there are isolated islands in the network. Since there is at most one substation node in each connected subgraph, it is easy to know that $C > N_{\mathrm{s}}$. As there is no connected loop in each connected subgraph, that is, the connected subgraph presents a tree structure, the number of closed branches in each subgraph satisfies the following equation.

$$\sum_{r=1}^{R_c} x_{cr} = Q_c - 1 \tag{12}$$

Where R_c is the number of branches in the cth connected subgraph; x_{cr} is the state of the rth branch in the cth subgraph.

The number of closed branches in the network is as follows:

$$\sum_{c=1}^{C}\sum_{r=1}^{R_c} x_{cr} = \sum_{c=1}^{C}(Q_c - 1) = N - C \tag{13}$$

Due to $C > N_{\mathrm{s}}$, the total number of closed branches in the network is $N\text{-}C < N\text{-}N_{\mathrm{s}}$, which is contradictory to condition 1. Therefore, the assumption is invalid, the network satisfies radial topology, and the sufficiency of proposition 1 is proved.

Proof of necessity: If a distribution network is radial, obviously there is no connected loop in the network, and condition 3 is true. In addition, connected subgraphs of the radial network are all tree-shaped, and the number of closed branches satisfies equation 12. Since each connected subgraph contains 1 substation, the number of connected subgraphs is equal to the number of substations, that is, $C = N_{\mathrm{s}}$. The total number of closed branches is shown in equation 14.

$$\sum_{c=1}^{C}\sum_{r=1}^{R_c} x_{cr} = \sum_{c=1}^{N_s}(Q_c - 1) = N - N_{\mathrm{s}} \tag{14}$$

Thus, condition 3 is also true, and the necessity of proposition 1 is proved.

B. Loop Search Algorithm for Distribution Networks

To apply the RCRM based on loop disconnection to represent the radial constraint of a distribution network, it is necessary to obtain all loops in the network first. Some researchers have proposed to get all the loops in a graph by combining the basic loops [9]. The so-called basic loop is the independent loop in the network. Different from the loop in graph theory, the loop in a distribution network also contains the path between two substation nodes. All loops in a distribution network can be obtained with similar ideas.

The basic loops of a distribution network can be obtained from the spanning forest of its topological graph. Each tree in the forest is required to contain a substation. The addition of any link branch in the forest produces a loop, and the collection of these loops forms a set of basic loops. Based on the depth-first traversal algorithm, the basic loops of the network can be obtained in the process of searching spanning-forest.

Arrange the branches of the distribution network in a certain order. And define a binary array of length B for each loop, each bit of which corresponds to a branch. For branches contained in the loop, the corresponding bits in the array are assigned 1, and the other bits are assigned 0. The combination method of two loops is to perform xor operation bit by bit on their binary arrays so that only their unique branches are retained in the new loop. The process of obtaining all loops by combination of basic loops is as follows:

- Initially add one basic loop to the loop set.
- Combine the remaining basic loop with all loops in the loop set successively, and add the basic loop and the newly obtained loops into the set.
- Arrange the branches in the loop according to the actual connection sequence. If the branches in the loop cannot be connected together, which indicates the loop does not exist, then delete the loop from the loop set.

Take the network shown in Fig. 1 as an example to illustrate the above combination process, where F1 and F2 are substation nodes, Z1-Z4 are load nodes, and number 1-7 represents the number and sequence of branches.

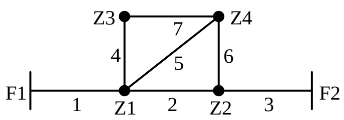

Fig. 1. A simple network

Assume that the basic loops in Fig. 1 are 1—2—3, 2—5—6 and 4—5—7, and their corresponding arrays are [1,1,1,0,0,0,0], [0,1,0,0,1,1,0] and [0,0,0,1,1,0,1], respectively. Initially add 1—2—3 to the loop set. First, combine 2—5—6 with loops in the set. Through this operation, array [1,0,1,0,1,1,0] is obtained, namely loop 1—5—6—3. Add the above two loops to the set. Then combine 4—5—7 with the loops in the set to get 1—2—3—4—5—7, 2—4—6—7 and 1—4—7—6—3, and add them to the set. Finally, arrange the branches of each loop in the set. The branches of loop 1—2—3—4—5—7 cannot be connected end to end, so delete it from the set.

IV. Comparison of Different RCRMs

This section compares the proposed model, spanning tree model, virtual demand model, and path-based model. The number of variables and constraint equations in different models are listed in TABLE I.

TABLE I. Comparison of different RCRMs mathematical models

Model	Number of variables	Number of constraints
model in this paper	B	L+1
spanning-tree model	3B	B+N
virtual demand model	2B	2B+N+1
path-based model	P	N+P-2N_s

The size relation between the number of branches and power supply paths is $B \le P$, and the equal sign is true only when there is no loop in the network. When the number of loops is large, the difference between them is particularly obvious. Therefore, compared with the existing models, the model in this paper has the least number of variables.

The virtual demand model has fewer variables than the spanning tree model, but it has more constraint equations.

In addition to the number of basic loops, the number of loops is also closely related to the combinability of basic loops. When the basic loops are not combinable, the constraint number of the model in this paper is far less than that of the spanning-tree model and virtual demand model. When the number of combinable basic loops is large, the total number of loops will increase rapidly, and the constraint number of the model in this paper may exceed the spanning-tree model and virtual demand model.

One loop can always correspond to multiple power supply paths, and branches not included in any loop can also constitute power supply paths, so the number of power supply paths is much larger than the number of loops. For example, in the IEEE test feeder of 4 nodes, 13 nodes, 34 nodes, and 123 nodes, the number of loops is 0, 0, 0 and 3 respectively, but the number of power supply paths reaches 3, 12, 33 and 380 respectively. Thus compared with the path-based model, the complexity of the model in this paper is always lower.

In summary, when the number of combinable loops in the distribution network is relatively small, it is appropriate to use the RCRM proposed in this paper. When the number of loops in the network is large, the spanning tree model or virtual demand model should be selected. When the problem to be analyzed is closely related to the power supply path, the path-based model can be adopted.

V. Service Restoration Reconfiguration

This paper takes the service restoration reconfiguration problem of the distribution network as an example to compare the computing performance of different RCRMs.

There are many optional load transfer schemes in service restoration and reconstruction. Different objective functions can obtain different reconstruction effects. In order to make the network structure changeless and extend the service life of switches, this paper takes the minimum number of switch operations as the goal, and the objective function is as follows:

$$\min\left(\sum_{T_b \in I_s} x_b + \sum_{T_b \in O_s} (1 - x_b)\right) \tag{15}$$

Where I_s is the set of branches where contact switches are located under normal operation; O_s is the set of branches where sectional switches are located; T_b is the bth branch in the network; The first summation formula represents the switching times of the contact switches; The second summation represents the switching times of the sectional switches.

The power flow constraints are as follows [8]:

$$\sum_{n_j \in \Gamma_i} f_{ij} = -u_i,\ i = 1, 2, \cdots, N - N_s \tag{16}$$

$$\sum_{n_j \in \Gamma_i} f_{ij} \le S_i,\ i = 1, 2, \cdots, N_s \tag{17}$$

Where f_{ij} is the power flow from n_i to n_j; u_i is the load capacity of the ith load node; S_i is the capacity of the ith substation node.

Constraints of feeders' transmission capacity are as follows:

$$-x_b F_b \le f_{ij} \le x_b F_b \tag{18}$$

Where F_b is the maximum carrying capacity of the bth branch.

The radial constraint of the distribution network is represented in different ways as introduced in Section II and III.

VI. Case Analysis

A. The Case and Testing Environment

In this paper, a 10 kV distribution network in Fuzhou, China is used to verify the effectiveness of the proposed RCRM and to compare the computing performance of different RCRMs. The distribution network is shown in Fig. 2.

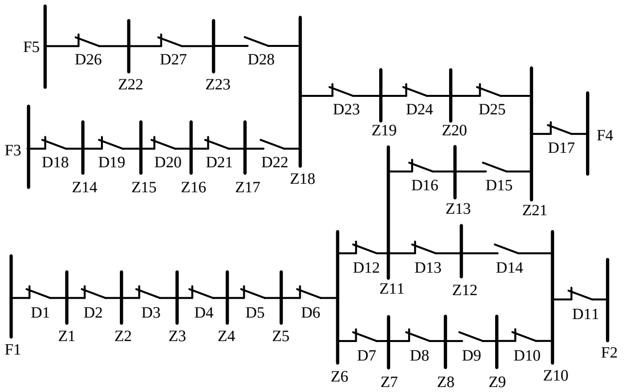

Fig. 2. Diagram of an actual distribution network

Where F1−F5 are the substation nodes; Z1−Z23 are the load nodes; D1−D28 are the switches, among which D9, D14, D15, D22, and D28 are contact switches, which are opened in normal operation mode, and the remaining 23 are sectional switches, which are closed in normal operation mode.

The capacity of each substation node and load node is listed in TABLE II.

TABLE II. Capacity of Each Substation Node and Load Node

Node	Capacity (MVA)	Node	Capacity (MVA)	Node	Capacity (MVA)
Z1	1.1	Z11	0	Z21	0.62
Z2	0.67	Z12	0.8	Z22	3.47
Z3	0.32	Z13	0.31	Z23	0.88
Z4	1.8	Z14	1.52	F1	8.712
Z5	0.16	Z15	1.26	F2	8.712
Z6	1.142	Z16	0.76	F3	8.712
Z7	0.1	Z17	0.4	F4	8.712
Z8	1.04	Z18	0.35	F5	8.712
Z9	0	Z19	0.85		
Z10	2.4	Z20	0.345		

The capacity of each branch is listed in TABLE III.

TABLE III. Capacity of Each Branch

Branch	Capacity (MVA)	Branch	Capacity (MVA)	Branch	Capacity (MVA)
D1	8.712	D11	8.712	D21	7.707
D2	8.712	D12	8.712	D22	7.707
D3	8.712	D13	7.326	D23	7.707
D4	8.712	D14	8.712	D24	8.712
D5	7.326	D15	8.712	D25	8.712
D6	7.326	D16	8.712	D26	8.712
D7	7.326	D17	8.712	D27	7.707
D8	7.326	D18	8.712	D28	7.707
D9	7.326	D19	7.707		
D10	7.326	D20	7.707		

Using CPLEX optimization software to solve the service restoration reconfiguration problem, and using Java language to write the program. The program runs on Windows 10 of 64 bits. The CPU model in the test is Intel Core i7-7700K, with 3.60GHz master frequency and 16GB memory.

B. Service Restoration Reconfiguration Analysis

Representing the radial constraint of the distribution network by the RCRM based on loop disconnection, and establishing the service restoration reconfiguration model of the distribution network described in Section IV.

Assume the substations F1−F5 fault successively, and calculate the scheme of network reconstruction respectively. The results are shown in TABLE IV.

TABLE IV. Calculation Results of Service Restoration Reconfiguration

Faulty Substation	Optimal Value	Action Switches	Number of constraints
F1	3	D15, D22, D25	F3, F4
F2	3	D6, D14, D15	F4
F3	1	D22	F4
F4	1	D22	F3
F5	1	D28	F4

It is obvious that the service restoration reconfiguration reconstruction schemes in TABLE IV can keep the distribution network to be radial, which verifies the correctness of the RCRM proposed in this paper.

C. Comparison of Computing Time of Different RCRMs

In the service restoration reconfiguration problem, using different RCRMs introduced in Section II and III to represent radial constraints of the distribution network. The number of variables and constraint equations in the established mathematical model are shown in Fig. 3.

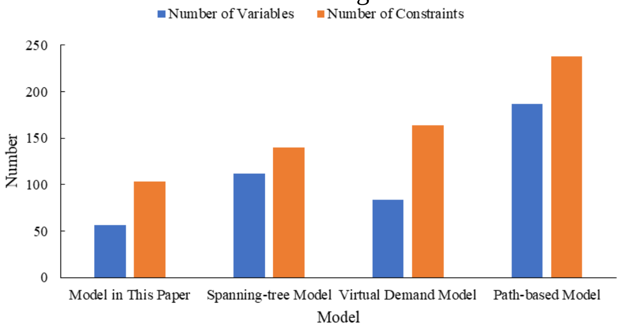

Fig. 3. Comparison of models based on different RCRMs

The number of loops, nodes, branches, and power supply paths in the distribution network shown in Fig. 2 is 18, 28, 28 and 187 respectively. The number of loops is far less than the number of nodes, branches, and power supply paths. Therefore, the number of variables and constraint equations required by the model proposed in this paper are significantly less than other models, and the complexity of the model based on loop disconnection is the lowest.

Assume the substations F1−F5 fault successively, and solve the service restoration reconfiguration models based on different RCRMs respectively. The computing time of different models is shown in TABLE V.

TABLE V. COMPUTING TIME OF DIFFERENT MODELS

Faulty Substation	**Computing Time (ms)**			
	Model in This Paper	***Spanning-tree Model***	***Virtual Demand Model***	***Path-based Model***
F1	12.5	14.7	16.7	26.6
F2	12.3	15.6	17.8	22.5
F3	7.2	8.2	10.6	18.6
F4	10.1	11.3	16.3	21.4
F5	7.4	8.7	9.8	16.6

In combination with TABLE V and Fig. 3, it can be seen that the complexity of the mathematical model has a certain correlation with computing efficiency. As analyzed in Section IV, for distribution networks with fewer loops, such as the one shown in Fig. 2, the computing performance of the model proposed in this paper is superior to other models. The path-based model is the most complex with the longest computing time. And the computing performance of the spanning-tree model is better than that of the virtual demand model.

VII. CONCLUSION

This paper proposes a RCRM for distribution networks based on loop disconnection and summarizes the existing RCRMs. The comparative analysis shows that the RCRM based on loop disconnection is applicable to the case where the number of loops in the distribution network is not too large. When the number of loops is large, the spanning tree model or virtual demand model should be selected, among which the spanning tree model is a better choice. When the problem to be analyzed is closely related to the power supply path and the number of loops in the network is small, the power supply path model can be adopted. Simulation analysis of the service restoration reconfiguration reconstruction problem of the distribution network verifies the above conclusions.

Our future work will focus on studying more RCRMs for distribution networks, so as to further reduce the complexity of the mathematical model and improve the solving efficiency of the distribution network optimization problems.

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REFERENCES

[1] B. Moradzadeh, G. Liu, and K. Tomsovic, "Robust reconfiguration of a distribution system," *Proceedings of the 50th Hawaii International Conference on System Sciences*, Hawai, USA, 2017, pp. 3222–3230.

[2] R. A. Jabr, R. Singh, and B. C. Pal, "Minimum loss network reconfiguration using mixed-integer convex programming," *IEEE Trans. on Power Syst.*, vol. 27, no. 2, pp. 1106–1115, May. 2012.

[3] H. Haghighat, and B. Zeng, "Distribution system reconfiguration under uncertain load and renewable generation," *IEEE Trans. on Power Syst.*, vol. 31, no. 4, pp. 2666–2675, Jul. 2016.

[4] M. Lavorato, J. F. Franco, M. J. Rider, and R. Romero, "Imposing radiality constraints in distributio system optimization problems," *IEEE Trans. on Power Syst.*, vol. 27, no. 1, pp. 172–180, Jan. 2012.

[5] N. G. Paterakis, A. Mazza, S. F. Santos, O. Erdinc, G. Chicco, A. G. Bakirtzis, et al., "Multi-objective reconfiguration of radial distribution systems using reliability indices," *IEEE Trans. on Power Syst.*, vol. 31, no. 2, pp. 1048–1062, Mar. 2016.

[6] F. Capitanescu, L. F. Ochoa, H. Margossian, and N. D. Hatziargyriou, "Assessing the potential of network reconfiguration to improve distributed generation hosting capacity in active distribution systems," *IEEE Trans. on Power Syst.*, vol. 30, no. 1, pp. 346–356, Jan. 2015.

[7] E. R. Ramos, A. G. Exposito, J. R. Santos, and F. L. Iborra, "Path-based distribution network modeling: application to reconfiguration for loss reduction," *IEEE Trans. on Power Syst.*, vol. 20, no. 2, pp. 556–564, May. 2005.

[8] H. Hong, Z. Hu, R. Guo, J. Ma, and J. Tian, "Directed graph-based distribution network reconfiguration for operation mode adjustment and service restoration considering distributed generation," *Journal of Modern Power Systems and Clean Energy*, vol. 5, no. 1, pp. 142–149, Jan. 2017.

[9] M. Safar, K. Alenzi, and S. Albehairy, "Counting cycles in an undirected graph using DFS-XOR algorithm," *First International Conference on Networked Digital Technologies*, Ostrava, Czech Republic, 2009, pp. 132–139.