An Improved Algorithm for State Estimator Based on Maximum Normal Measurement Rate

Bin Wang, Guangyu He, *Member, IEEE*, and Shufeng Dong

***Abstract*—Previous work has shown that robust state estimator based on maximum normal measurement rate is accurate and reliable. However, in the existing algorithm, the optimal solutions cannot be obtained due to the approximation of evaluation function. Besides, there are no noise-filtering effects on normal measurements. To deal with these two problems, this letter presents an improved algorithm to get a more accurate solution with higher normal measurement rate. Numerical tests on different systems show the proposed algorithm is efficient.**

***Index Terms*—Normal measurement rate, robust state estimation, uncertainty in measurement.**

I. Introduction

Robust state estimator, which is capable of bad data rejection in power system analysis, has been studied since decades ago. Some novel robust state estimator based on measurement uncertainty has been proposed in recent years. Gastoni, Granelli, and Montagna proposed a robust estimator based on the maximum agreement between measurements [1]. Al-Othman and Irving presented a new robust estimator based on maximum constraints satisfaction (MCS) of uncertain measurements [2]. Irving then formulated state estimation as a mixed integer programming problem and aimed at maximizing the number of estimated measurements that lie within the tolerances [3].

In our previous work, a robust state estimator based on maximum normal measurement rate (MNMR) is proposed [4]. The relative deviation for measurement i is defined as $d_i = (h_i(\boldsymbol{x}) - Z_i)/U_i$, where $h_i(\boldsymbol{x})$ is the measure function, Z_i is the measured value, and U_i is the expanded uncertainty of measurement i. The evaluation function $g(d_i)$ is as follows:

$$g(d_i) = \begin{cases} 0 & |d_i| \le 1 \\ 1 & |d_i| > 1. \end{cases} \tag{1}$$

If $g(d_i)$ is 0, measurement i is normal; otherwise, it is abnormal. MNMR aims at finding a solution that maximizes the number of normal measurements. In order to solve this problem, which is inherently combinational, an approximated evaluation function is proposed to model the problem as nonlinear optimal programming, which leads to an approximate optimal solution. Besides, the noises on normal measurements are not filtered since the evaluation value of normal measurement is always zero.

To deal with these two problems, this letter proposes an improved algorithm which consists of three steps. The first step is the approximate optimization step, in which a nonlinear optimal problem is solved to obtain an approximate optimal solution and a normal measurement set [4]. The second step is the consistency checking step, in which each abnormal measurement is checked whether there is a solution to make it consistent with the measurements in the existing normal measurement set. The third step is the noise filtering step, in which weighted least square estimation with the inequality constraints of all normal measurements are solved to filter the noises on normal measurements.

Numerical tests on different systems show that a more accurate solution with higher normal measurement rate can be obtained by the proposed algorithm.

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B. Wang and G. He are with Tsinghua University, Beijing, China.

S. Dong is with Zhejiang University, Hangzhou, China (e-mail: dongshufeng@zju.edu.cn).

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II. Improved Algorithm for MNMR

A. Approximate Optimization Step

The details of this step are documented in [4] and will be reviewed briefly below. An approximate evaluation function $f(d_i)$ is defined as follows:

$$f(d_i) = \delta(d_i) + \delta(-d_i) \tag{2}$$

where $\delta(d_i)$ is a sigmoid function, defined as follows:

$$\delta(d_i) = \frac{1}{\left(1 + e^{-k(ad_i+b)}\right)}. \tag{3}$$

MNMR is modeled as a nonlinear optimal problem:

$$\begin{aligned} &\min \sum_{i=1}^{m} f(d_i) \\ &s.t.\ d_i = \left(h_i(\boldsymbol{x}) - Z_i\right)/U_i, \quad \forall i = 1, 2, \cdots, m \\ &\qquad \boldsymbol{g}(\boldsymbol{x}) = 0, \boldsymbol{l}(\boldsymbol{x}) \le 0 \end{aligned} \tag{4}$$

where $\boldsymbol{g}(\boldsymbol{x}) = 0$ represents power flow constraints and $\boldsymbol{l}(\boldsymbol{x}) \le 0$ represents physical constraints in practical operation. By solving (4), an approximate optimal solution $\boldsymbol{x}^{(0)}$ and a normal measurement index set $\mathbf{N}^{(0)}$ are obtained.

B. Consistency Checking Step

Define the set of all feasible state variables as

$$\mathbf{X}^{(0)} = \left\{ \boldsymbol{x} | \boldsymbol{g}(\boldsymbol{x}) = 0, \boldsymbol{l}(\boldsymbol{x}) \le 0, \left| \frac{(h_j(x) - Z_j)}{U_j} \right| \le 1, \quad \forall j \in \mathbf{N}^{(0)} \right\}. \tag{5}$$

For an abnormal measurement$i$, compute its possible range $[Z_i^L, Z_i^U]$ in set $\mathbf{X}^{(0)}$ by solving the following two optimal problems with $\boldsymbol{x}^{(0)}$ regarded as the initial states:

$$Z_i^L = \min\nolimits_{\boldsymbol{x} \in \mathbf{X}^{(0)}} h_i(\boldsymbol{x}) \tag{6}$$

$$Z_i^U = \max\nolimits_{\boldsymbol{x} \in \mathbf{X}^{(0)}} h_i(\boldsymbol{x}). \tag{7}$$

TABLE I
ABNORMAL MEASUREMENTS IN BOTH MNMR AND IMPROVED MNMR

System	*Actual Bad Data*	*Abnormal measurements in MNMR*	*Abnormal measurements in Improved MNMR*
IEEE 14	$P_{12\text{-}13}$,$Q_{13\text{-}12}$	$P_{12\text{-}13}$,$Q_{13\text{-}12}$, $\mathbf{Q_{12\text{-}13}}$	$P_{12\text{-}13}$,$Q_{13\text{-}12}$
IEEE 30	V_{19},P_{19},Q_{19},$P_{27\text{-}25}$,$Q_{27\text{-}25}$,$P_{12\text{-}16}$,$Q_{12\text{-}16}$,$\mathbf{Q_{16\text{-}12}}$,$\mathbf{P_{12\text{-}15}}$	V_{19},P_{19},Q_{19},$P_{27\text{-}25}$, $Q_{27\text{-}25}$,$P_{12\text{-}16}$,$Q_{12\text{-}16}$, $\mathbf{P_{21\text{-}22}}$, $\mathbf{Q_{13\text{-}12}}$, $\mathbf{Q_{16}}$, $\mathbf{Q_{27}}$, $\mathbf{P_{22\text{-}21}}$	V_{19},P_{19},Q_{19},$P_{27\text{-}25}$,$Q_{27\text{-}25}$,$P_{12\text{-}16}$,$Q_{12\text{-}16}$, $\mathbf{Q_{16}}$
IEEE 118	V_{35},P_{35},$P_{30\text{-}17}$,$Q_{30\text{-}17}$,$Q_{28\text{-}29}$,$P_{29\text{-}28}$,$Q_{29\text{-}28}$,$P_{17\text{-}30}$,Q_{35}	V_{35},P_{35},$P_{30\text{-}17}$,$Q_{30\text{-}17}$,$Q_{28\text{-}29}$,$P_{29\text{-}28}$,$Q_{29\text{-}28}$,$P_{17\text{-}30}$,$\mathbf{Q_{28}}$,$\mathbf{Q_{29}}$,$\mathbf{Q_{31}}$,$\mathbf{Q_{37\text{-}35}}$	V_{35},P_{35},$P_{30\text{-}17}$,$Q_{30\text{-}17}$,$Q_{28\text{-}29}$,$P_{29\text{-}28}$,$Q_{29\text{-}28}$,$P_{17\text{-}30}$,$\mathbf{Q_{37\text{-}35}}$

If $[Z_i^L, Z_i^U] \cap [Z_i - U_i, Z_i + U_i] \neq \emptyset$, it means there exists a solution $\boldsymbol{x}^{(i)} \in \mathbf{X}$ which makes measurement i consistent with all measurements in $\mathbf{N}^{(0)}$, then add i to $\mathbf{N}^{(0)}$ and form a new set $\mathbf{N}^{(1)}$; otherwise, measurement i will always be abnormal.

Check every abnormal measurement in the same way to see whether it can be added to normal measurement set. A new normal measurement set $\mathbf{N}$ is obtained, which is supposed to be larger than $\mathbf{N}^{(0)}$. The checking process starts from the measurement with smaller residual to the one with larger residual. Since there are few abnormal measurements after the first step, the checking process in this step is quite efficient.

C. Noise Filtering Step

To filter the noise on normal measurements, a weighted least square state estimation with inequality constraints of all normal measurements are formulated as follows:

$$\begin{aligned} \min \sum_{i=1}^{m} w_i \left(h_i(\boldsymbol{x}) - Z_i\right)^2 \\ s.t. \left|\frac{(h_j(\boldsymbol{x}) - Z_j)}{U_j}\right| \leq 1, \quad \forall j \in \mathbf{N} \\ \boldsymbol{g}(\boldsymbol{x}) = 0,\ \boldsymbol{l}(\boldsymbol{x}) \leq 0 \end{aligned} \tag{8}$$

where $w_i = (1/U_i)^2$. By solving (8), a more accurate solution $\boldsymbol{x}$ with higher normal measurement rate is obtained.

III. NUMERICAL RESULTS

Firstly, three IEEE standard systems are tested. Full measurements are configured in each system and Gaussian noise with zero mean and variance of 2% of the nominal value is added to the measured value. If a measurement is regarded as bad data, an extra deviation of 20% of the nominal value is added to the measured value.

The proposed algorithm, denoted as Improved MNMR, is compared to the existing algorithm [4], denoted as MNMR. All parameters relevant to the optimal model (4) are chosen the same as [4]. Table I shows the abnormal measurements sets obtained by both algorithms. We can see that in all cases, the improved MNMR can get a solution with higher normal measurement rate than MNMR, due to the consistency checking step. Table II shows the index $\mathbf{E_v}$ and $\mathbf{E}_{\boldsymbol{\theta}}$ defined in [4], which represents the distance between the estimated state and the true state. We can see that the improved MNMR can get a more accurate solution than MNMR, due to the noise filtering step.

Secondly, a real 272-bus system which is located in the east of China, denoted as system SH, is tested. The uncertainty interval of each measurement is determined according to the evaluation indices proposed by the power company. MNMR gets a solution with 104 abnormal measurements in all 1797 measurements, while the improved MNMR gets a solution with only 73 abnormal measurements, which improve the normal measurement rate from 94.21% to 95.94%.

TABLE II
ACCURATE COMPARISON BETWEEN MNMR AND IMPROVED MNMR

System	MNMR		Improved MNMR	
	$\mathbf{E_v}$	$\mathbf{E_\theta}$	$\mathbf{E_v}$	$\mathbf{E_\theta}$
IEEE 14	1.0757E-5	9.7928E-4	1.4569E-6	1.8983E-4
IEEE 30	4.6283E-5	0.01305	1.2527 E-5	0.001810
IEEE 118	1.2627E-7	0.005764	6.0065E-8	0.002446

TABLE III
COMPUTATION TIME FOR THE LAST TWO STEPS IN IMPROVED MNMR

System	IEEE 14	IEEE 30	IEEE 118	SH
Time/s	0.332	1.106	6.323	76.295

The proposed algorithm is implemented with JAVA, while all tests are carried on a personal computer with 2.0-GHz Intel (R) Core CPU and 2 GB of RAM. Table III shows the computation time of the added two steps in the improved MNMR. Since the number of abnormal measurements is usually small, the compute time added is affordable.

IV. CONCLUSIONS

This letter has presented an improved algorithm for robust state estimation based on maximum normal measurement rate. A consistency checking step is added to obtain a solution with higher normal measurement rate, which is more reliable. Then a noise filtering step is added to get a more accurate solution. Since the number of abnormal measurements is usually small, the compute time added is affordable.

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