Robust State Estimator Based on Maximum Normal Measurement Rate

Guangyu He, *Member, IEEE*, Shufeng Dong, Junjian Qi, and Yating Wang

***Abstract*—In this paper, the concept of normal measurement rate (NMR) is defined based on the theory of uncertainty in measurement and a robust state estimator named maximum normal measurement rate (MNMR) estimator is proposed. Comparison is made between the MNMR estimator and other robust estimators. The robustness and precision of this estimator is tested with the IEEE 14-bus, 30-bus, and 118-bus systems. Simulation results show that the MNMR estimator is effective in identifying bad data and is also time efficient.**

***Index Terms*—Normal measurement rate, robust state estimation, uncertainty in measurement.**

I. Introduction

STATE estimation is the basis of power system analysis and control. Weighted least squares estimate has been discussed in many literatures, such as [1]–[3], and has been applied in many fields very successfully. It exhibits efficient filtering capability when the errors are Gaussian, but is very sensitive to bad data and may cause very poor estimates. In order to guarantee the estimator to exhibit stable behavior under deviation from the assumptions on which they are based, various robust estimators have been proposed, such as M-estimator [4]–[14], GM-estimator [6], [13], [15], and high breakdown point estimator [16]–[19].

M-estimator, generalized maximum likelihood estimator, mainly include weighted least absolute value (WLAV) estimator [7]–[11], quadratic-linear (QL) estimator [6], [13], [15], and quadratic-constant (QC) estimator [6], [12], [14].

GM-estimator, generalized M-estimator, mainly include Mallows-type GM-estimator [13] and Schweppe-type GM-estimator [6], [13], [15].

High breakdown point estimator, estimators with high breakdown point, mainly include least median of squares (LMS) estimator and least trimmed squares (LTS) estimator [16]–[19].

Apart from the above-mentioned methods, some novel robust estimators have also been proposed in recent years. Gastoni, Granelli, and Montagna proposed a robust estimator based on the maximum agreement between measurements [20]. Al-Othman and Irving presented a new robust estimator based on maximum constraints satisfaction (MCS) of uncertain measurements [21]. In [21], they considered the uncertainty in the measurement and modeled the uncertainty via deterministic upper and lower bounds on measurement errors. Irving then formulated state estimation as a mixed integer programming problem and aimed at maximizing the number of estimated measurements that lie within tolerance [22].

However, there are two limitations within the MCS estimator and the latter mixed integer programming method. Firstly, an estimate is found which fits the stated tolerance ranges and has little or no noise filtering effect among the good measurements. Secondly, they both require a heavy computational time burden and are thus difficult to be applied to large-scale power systems, although [22] has pointed out that mixed integer programs with thousands of variables can now be solved routinely in less than one minute.

In order to solve these problems, a new robust estimator based on the maximum normal measurement rate (MNMR) is proposed in this paper. Section II introduces the concept of uncertainty in measurements and defines normal measure point and normal measurement rate. Section III presents the problem formulation of the MNMR estimator. Sections IV and V compare the proposed method with non-quadratic methods and the MCS estimator. Simulation results are presented in Section VI.

II. Concept of Normal Measurement Rate

In 1993, seven international organizations issued "Guide to the expression of uncertainty in measurement" [23], in which they defined uncertainty in measurement as parameter associated with the result of a measurement, which characterizes the dispersion of the values that could reasonably be attributed to the measurand. This concept reflects the possible distribution range of the error and can be considered as the error limit under certain confidence probability.

There are two kinds of uncertainties: standard uncertainty and expanded uncertainty.

For standard uncertainty, uncertainty of the result of a measurement is expressed as a standard deviation. From its definition [23], for measure point i, Z_i is the measurement, the standard uncertainty is denoted by u, the probability that the true value $\bar{Z}_i$ lies in the interval $[Z_i - u, Z_i + u]$ is taken as p, as is shown in (1):

$$P(|\bar{Z}_i - Z_i| \leq u) = p, \forall i. \quad (1)$$

If the uncertainty follows normal distribution, p will be 68.3%.

Expanded uncertainty is interpreted as defining an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could

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The authors are with the State Key Laboratory of Power Systems, Department of Electrical Engineering, Tsinghua University, Beijing 100084, China (e-mail: gyhe@mail.tsinghua.edu.cn; dongshufeng@gmail.com; qjj08@mails.tsinghua.edu.cn; wyt1987@gmail.com).

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reasonably be attributed to the measurand. The expanded uncertainty U is obtained by multiplying the standard uncertainty u by a coverage factor k:

$$U = ku \text{ (k is a natural number)}. \quad (2)$$

According to the definition of expanded uncertainty, for measure point i, Z_i is the measurement, the expanded uncertainty is denoted by U_i, the probability that the true value lies in the interval $[Z_i - U_i, Z_i + U_i]$ is taken as p, as is shown in (3):

$$P(|\bar{Z}_i - Z_i| \le U_i) = p, \forall i. \quad (3)$$

If $k = 3$ and the uncertainty follows normal distribution, p will be 99.7%.

It needs to be pointed out that expanded uncertainty is independent of specific distribution and it is not assumed to follow normal distribution.

In practical applications, p is generally taken as a value significantly greater than 0.5, such as 0.997 or 0.945, and then the corresponding U_i can be determined.

Since the true value lies in the interval $[Z_i - U_i, Z_i + U_i]$ with a large probability, its potential estimation value also lies in the interval with a large probability. In light of this, we define normal measure point, abnormal measure point, and normal measurement rate.

Definition 1: Given the expanded uncertainty of measure point i under the confidence probability p as U_i, x is the state estimation result, $h_i(x)$ is the ith measurement function, if (4) is satisfied, then measure point i is a **normal measure point** under state estimation solution x. Otherwise, it is an **abnormal measure point**:

$$|h_i(x) - Z_i| \le U_i. \quad (4)$$

Definition 2: Suppose the number of measure points in the system is m, the confidence probability for all the measure points is p, the expanded uncertainty for measure point i is U_i, and the number of normal measure points under the state estimation solution x is n, then the normal measurement rate (NMR) under the state estimation solution x is defined as follows:

$$\eta = n/m * 100\%. \quad (5)$$

III. Maximum Normal Measurement Rate Estimator

The MCS estimator [21] models uncertainty in the measurement via deterministic upper and lower bounds on the measurement errors and aims at finding a solution satisfying most inequality constraints.

Reference [21] has pointed out that a solution in the region established by the uncertain bounds of measurements with gross errors can never have a maximum number of satisfied constraints that exceeds that of the region established by the good measurements and thus the MCS estimator guarantees a robust solution.

Based on this idea, we propose a robust estimator named MNMR estimator. It aims at finding a solution that makes the number of normal measure points maximum and thus the normal measurement rate. To achieve this, we first define the NMR evaluation index.

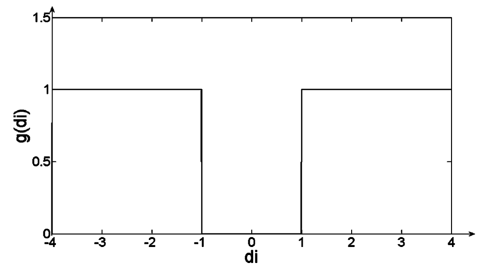

Fig. 1. Figure of $g(d_i)$.

A. Ideal NMR Evaluation Index

Given a system state x, the relative deviation at measure point i can be defined as (6):

$$d_i = (h_i(x) - Z_i)/U_i \quad (6)$$

where $Z_i, h_i(x)$, and U_i are measurement, measurement function, and the expanded uncertainty at measure point i. U_i corresponds to a confidence probability p.

The ideal evaluation function of measure point i is constructed as follows:

$$g(d_i) = \begin{cases} 0 & |d_i| \le 1 \\ 1 & |d_i| > 1. \end{cases} \quad (7)$$

If measure point i is normal ($|d_i| \le 1$), $g(d_i)$ is 0; otherwise, $g(d_i)$ is 1. $g(d_i)$ is illustrated in Fig. 1.

We define $\sum_{i=1}^{m} g(d_i)$, the summed evaluation function of all measure points, as the ideal NMR evaluation index.

Obviously, a solution with the smallest ideal NMR evaluation index has the highest NMR. So maximizing NMR can be converted into minimizing the ideal NMR evaluation index.

B. NMR Evaluation Index in Practical Applications

In practical applications, measure point i is not considered as abnormal if $|d_i|$ is only slightly greater than 1. Only if $|d_i|$ is considerably greater than 1, can the measure point be considered as abnormal. Thus, the evaluation function $f(d_i)$ should be approximate 0 when $|d_i|$ is less than or equal to 1 while should be approximate 1 only when $|d_i|$ is greater than λ (λ is a positive constant considerably greater than 1, such as 2).

In light of this, we redefine the concept of normal and abnormal measure point in Definition 1 and also define the concept of suspicious measure point.

For measure point i, if $|h_i(x) - Z_i| \le U_i$ or $|d_i| \le 1$, it is a normal measure point; if $|h_i(x) - Z_i| \ge \lambda U_i$ or $|d_i| \ge \lambda$, it is an abnormal measure point; if $U_i < |h_i(x) - Z_i| < \lambda U_i$ or $1 < |d_i| < \lambda$, it is a suspicious measure point.

In this paper, the NMR evaluation function $f(d_i)$ is defined as follows:

$$f(d_i) = \delta(d_i) + \delta(-d_i) \quad (8)$$

where

$$\delta(d_i) = \frac{1}{1 + e^{-k(ad_i + b)}}. \quad (9)$$

TABLE I
VALUES OF FUNCTION $f(d_i)$

k	$f(\pm1)$	$f(\pm\lambda)$
1	0.2869	0.7335
1.5	0.1849	0.8177
2	0.1195	0.8808
2.5	0.0759	0.9241
3	0.0474	0.9526
3.5	0.0293	0.9707
4	0.0180	0.9820

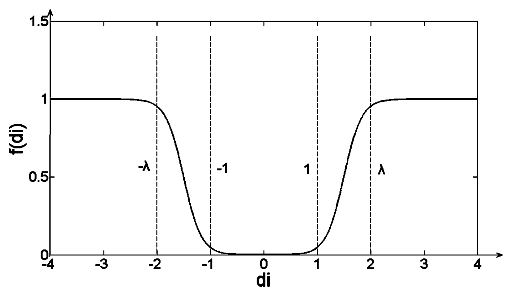

Fig. 2. Figure of $f(d_i)$ ($\lambda = 2$).

The $\delta(d_i)$ in (9) is actually a sigmoid function.

In order to make $f(\pm\lambda) \approx 1$ and $f(\pm1) \approx 0$, we select a and b in the following way:

$$a = \frac{2}{\lambda - 1} \tag{10}$$

$$b = -1 - \frac{2}{\lambda - 1}. \tag{11}$$

Therefore

$$\delta(d_i) = \frac{1}{1 + e^{\frac{2k}{\lambda-1}\left(-d_i + \left(1 + \frac{\lambda-1}{2}\right)\right)}}.$$

The NMR evaluation index is correspondingly defined as the summed evaluation function of all measure points $\sum_{i=1}^{m} f(d_i)$.

The values of $f(\pm1)$ and $f(\pm\lambda)$ as k changes are shown in Table I. It can be seen that when k gets greater, $f(\pm1)$ is more approximate to 0, $f(\pm\lambda)$ is more approximate to 1, and thus $f(d_i)$ is more approximate to $g(d_i)$.

However, when k is too great, it might cause difficulty in the convergence of the optimization problem with $\sum_{i=1}^{m} f(d_i)$ as objective function. Therefore, k is taken as 3 in this paper.

Then the $\delta(d_i)$ we really use is as (12):

$$\delta(d_i) = \frac{1}{1 + e^{\frac{6}{\lambda-1}\left(-d_i + \left(1 + \frac{\lambda-1}{2}\right)\right)}}. \tag{12}$$

In this case, when λ is 2, $f(d_i)$ can be illustrated in Fig. 2. It can be seen that $f(d_i)$ is similar to $g(d_i)$ but is continuous and differentiable at every point.

C. Mathematical Model of the Proposed Method

Since $\sum_{i=1}^{m} f(d_i)$ is approximate to abnormal measure points, to find a system state with the maximum number of normal measure points is to a large extent equivalent to find a system state with the minimal $\sum_{i=1}^{m} f(d_i)$. Therefore, the objective function of the proposed estimator is to minimize $\sum_{i=1}^{m} f(d_i)$ for a given solution x, as is shown in (13):

$$\min_x \sum_{i=1}^{m} f(d_i). \tag{13}$$

In practical power system operation, the system state should satisfy power flow constrains and other physical constrains, such as the limit of the outputs of generators. Considering these constraints, we construct the following state estimation model:

$$\begin{aligned} &\min_x \sum_{i=1}^{m} f(d_i) \\ \text{s.t.}\quad & d_i = (h_i(x) - Z_i)/U_i, \forall i = 1, \ldots, m \\ & g(x) = 0 \\ & l(x) \le 0 \end{aligned} \tag{14}$$

where $g(x) = 0$ represents power flow constrains and $l(x) \le 0$ represents physical constrains in practical operation.

D. Algorithm

Substituting d_i in (1) into the objective function, we have the following model:

$$\begin{aligned} &\min_x \sum_{i=1}^{m} f((h_i(x) - Z_i)/U_i) \\ \text{s.t.}\quad & g(x) = 0 \\ & l(x) \le 0. \end{aligned} \tag{15}$$

This optimization problem can be solved with many methods, among which modern interior point method [24] has many advantages, such as good convergence property, high computation speed, and no significant increase of computation time with the increased scale [25], [26].

We have successfully solved the proposed optimization problem with IPOPT (**I**nterior **P**oint **OPT**imizer) [27], which implements a primal-dual interior point algorithm. It has high computation speed and good convergence property, which allow us to apply the proposed state estimator to online engineering.

What needs to be pointed out is that the proposed optimization model is nonconvex. Modern interior point method cannot guarantee to find the global optimum of a nonconvex problem and its solution is influenced by the start values.

In order to avoid the solution converging to local optimum to the greatest extent, we perform a two-stage state estimation.

1) First Stage Estimation: In the first stage estimation, the initial state is a flat start.

By calculating the second-order derivation of (12), we can get the positive inflection point of the evaluation function (denoted by I), which actually only depends on the value of λ and can be written as

$$I = \frac{\lambda + 1}{2}. \tag{16}$$

When the parameter λ of the MNMR estimator is set to be a large enough positive number, the absolute value of the relative deviation defined in (6) will be less than or equal to I for all measure points under the initial state.

In this case, the evaluation function really used is actually a convex function of the relative deviation, which makes it have similar characteristics as the cost function of WLS and thus makes the solving of the proposed optimization problem with modern interior point method much easier.

In reality, there is no need or even inappropriate in some cases to make the absolute value of the relative deviation less than or equal to I for all measure points. Too large λ in the first stage estimation will make the first stage estimation almost entirely lose bad data identification property. Therefore, λ in the first stage estimation should be as small as possible as long as it guarantees that the absolute value of the relative deviation is greater than I for only a small proportion of measure points. In this paper, we choose the λ that makes the absolute value of the relative deviation greater than I for 10% of the measure points.

Specifically, we first calculate the relative deviation for all measure points when the system state x in (15) is a flat start and then find the qth largest relative deviation where q is 10% of the number of measure points. When 10% of the number of measure points is not an integer, it is rounded up and assigned to q. Finally, we choose λ as two times of the qth largest relative deviation minus 1 considering the relationship between the positive inflection point and λ shown in (16).

2) Second Stage Estimation: In the second stage estimation, the MNMR estimator takes the solution of the first stage estimation as its initial state and chooses much smaller λ, such as 2, so as to guarantee the bad data identification property of the estimator and thus to exactly find the bad data among measurements.

IV. Relationship and Difference Between the Proposed Method and Non-Quadratic Criteria

As other non-quadratic criteria [5]–[14], especially as QC method [6], [12], [14], the proposed evaluation function is also the function of residuals and also assigns less weight to large residue terms, which enables the proposed estimator less sensitive to bad data and at the same time makes the optimization problem more difficult to solve. Despite these similarities, there are also some distinct differences between our method and non-quadratic criteria.

1) The evaluation function in this paper is not only the function of residuals, but also the function of uncertainty in measurement. It is the uncertainty in measurement that has provided additional and valuable information about measurements and thus has helped to improve the performance of the estimator.
2) For non-quadratic criteria, when the residual is small enough, the cost function is usually chosen as quadratic function of the residual. However, in our method, we do not aim at achieving an exact fit of measurements. Since uncertainties inevitably exists in the measurements, it is more reasonable to make the estimated value only within a tolerance on the measurement.

V. Comparison With the Maximum Constraints Satisfaction Estimator

The MCS estimator and the MNMR estimator both take into account the uncertainty in measurements to realize a more robust state estimation. In reality, the MNMR estimator can be considered as an approximation of the MCS estimator in some sense, and thus has a similar bad data identification property as the MCS estimator.

However, there are also at least four differences between the MNMR estimator and the MCS estimator.

Firstly, different from the deterministic upper and lower bounds adopted by the MCS estimator, the uncertainty in measurement used in the MNMR estimator comes from [23] and corresponds to the same confidence probability for all measure points.

Secondly, the evaluation function we actually use allows the MNMR estimator to have some noise filtering effect among the good measurements because we introduce the concept of suspicious measure point. Thus, the estimation results are more precise than the MCS estimator.

Thirdly, state estimation can be solved much more efficiently in our method than the MCS estimator by using the modern interior point algorithm to solve a nonlinear programming problem. This allows the application to large-scale power systems.

Fourthly, the MCS estimator applies genetic algorithm to search the whole search space to find a global optimum, while modern interior point method cannot always find the global optimum in our method because of the nonconvex characteristic of the MNMR estimator. However, after utilizing a first stage estimation to provide initial state for the MNMR estimator, this problem can be solved to a large extent.

VI. Simulation Results

In this section, we test the MNMR estimator with IEEE 14-bus, 30-bus, and 118-bus systems, compare the MNMR estimator with the QC estimator, and also discuss the influence of the parameters of the MNMR estimator on state estimation results. All tests are carried out on a 3.0-GHz Inter(R) Core based personal computer and the proposed method is implemented with JAVA by calling IPOPT to solve the nonlinear programming problem.

Gaussian noise with zero mean and variance of 2% of the meter reading is added to the measurements.

A. IEEE AC Example

In this section, the MNMR estimator is applied to the IEEE 14-bus, 30-bus, and 118-bus systems. In all of the following cases, the expanded uncertainty U_i in (15) is taken as 3 times of the standard deviation of the measurements. In the first stage estimation, the initial state is a flat start.

We choose λ in the first stage estimation by adopting the method mentioned above and their values are shown in Table II, where *ratio* is the proportion of measure points for which the absolute value of the relative deviation is greater than the positive inflection point when the system state x in (15) is the initial state.

In the second stage estimation, λ is set to be 2 and the solution of the first stage estimation is used as the initial state.

As [20], the set of conforming multiple bad data shown in Table III was tested for the IEEE 14-bus system. The measurements are the same as [20]. Reference [20] has pointed out that

TABLE II
λ IN THE FIRST STAGE ESTIMATION FOR DIFFERENT TEST SYSTEMS

Test system	λ (First stage)	ratio
IEEE 14 bus	54.0595	0.1067
IEEE 30 bus	39.2948	0.1034
IEEE 118 bus	82.3761	0.1004

TABLE III
CONFORMING BAD DATA FOR THE IEEE 14-BUS SYSTEM

Data type	Good meas. (p.u.)	Bad data (p.u.)
$P_{1\text{-}2}$	1.5859	0.8500
$Q_{1\text{-}2}$	-0.2110	0.0000
P_1	2.3417	1.6200
Q_1	-0.1703	-0.0565

TABLE IV
ABNORMAL MEASURE POINTS IDENTIFIED BY THE SECOND STAGE ESTIMATION FOR IEEE 14-BUS SYSTEM

Data type	True value	Measurements	MNMR
$P_{1\text{-}2}$	1.5680	0.8500	1.5582
$Q_{1\text{-}2}$	-0.2039	0.0000	-0.2044
P_1	2.3235	1.6200	2.3101
Q_1	-0.1676	-0.0565	-0.1688

the largest normalized residual (LNR) method fails in this case because of the conforming nature of the four bad data.

In the first stage estimation, 23 measure points as suspicious measure points, and the others are all normal. In the second stage estimation, exactly 4 bad data are identified as abnormal measure points and all the others are normal. The abnormal measure points are listed in Table IV.

The MNMR estimator is also applied to the IEEE 30-bus system. The measurements are the same with [28]. We add 6 bad data to the measurements, as are shown in Table V. The flow measurements on 1–2 and the injection measurement at bus 1 are interacting conforming bad data, and the flow measurements on 24–25 and the injection measurement at bus 29 are noninteracting bad data [28].

The first stage estimation identified 3 abnormal measure points and 33 suspicious measure points. In the second stage estimation, exactly 6 bad data are identified as abnormal points and all the other points are normal. The abnormal measure points are listed in Table VI.

We also test conforming bad data with reference to the IEEE 118-bus system. The bad data are listed in Table VII.

In the first stage estimation, 24 measure points are identified as suspicious and all the others are normal. In the second stage estimation, exactly 4 bad data are identified as abnormal and all the others are normal. The abnormal measure points are listed in Table VIII.

TABLE V
BAD DATA FOR THE IEEE 30-BUS SYSTEM

Data type	Good meas. (p.u.)	Bad data (p.u.)
$P_{1\text{-}2}$	1.7786	0.0000
$Q_{1\text{-}2}$	-0.2562	0.2200
P_1	2.5825	0.0000
$P_{24\text{-}25}$	-0.0127	0.2000
P_{29}	-0.0244	-0.1200
Q_{29}	-0.0089	-0.1200

TABLE VI
ABNORMAL MEASURE POINTS IDENTIFIED BY THE SECOND STAGE ESTIMATION FOR IEEE 30-BUS SYSTEM

Data type	True value	Measurements	MNMR
$P_{1\text{-}2}$	1.7795	0.0000	1.7891
$Q_{1\text{-}2}$	-0.2578	0.2200	-0.2576
P_1	2.6103	0.0000	2.6257
$P_{24\text{-}25}$	-0.0123	0.2000	-0.0107
P_{29}	-0.0240	-0.1200	-0.0225
Q_{29}	-0.0090	-0.1200	-0.0088

TABLE VII
CONFORMING BAD DATA FOR THE IEEE 118-BUS SYSTEM

Data type	Good meas. (p.u.)	Bad data (p.u.)
$P_{1\text{-}2}$	-0.1297	-0.0620
$Q_{1\text{-}2}$	-0.1314	0.0000
P_1	-0.5169	-0.2550
Q_1	-0.3074	-0.1500

TABLE VIII
ABNORMAL MEASURE POINTS IDENTIFIED BY THE SECOND STAGE ESTIMATION FOR IEEE 118-BUS SYSTEM

Data type	True value	Measurements	MNMR
$P_{1\text{-}2}$	-0.1240	-0.0620	-0.1237
$Q_{1\text{-}2}$	-0.1265	0.0000	-0.1306
P_1	-0.5147	-0.2550	-0.5115
Q_1	-0.3027	-0.1500	-0.3022

TABLE IX
TIMING RESULTS FOR THE THREE IEEE TEST SYSTEMS

Test system	Time for choosing λ	Iterations		Time	
		First stage	Second stage	First stage	Second stage
14 bus	15ms	32	37	125ms	109ms
30 bus	16ms	34	52	188ms	203ms
118 bus	16ms	92	86	4188ms	4250ms

TABLE X
COMPARISON BETWEEN MNMR ESTIMATOR AND QC ESTIMATOR FOR THE 4 BAD DATA CASE OF IEEE 14-BUS SYSTEM

Estimator	n_{bad}	E_V	E_θ
MNMR	4(4bad)	7.5864×10^{-7}	0.0020
QC	5(4bad+1good)	2.9884×10^{-6}	4.2648×10^{-4}

TABLE XI
COMPARISON BETWEEN MNMR ESTIMATOR AND QC ESTIMATOR FOR THE 6 BAD DATA CASE OF IEEE 30-BUS SYSTEM

Estimator	n_{bad}	E_V	E_θ
MNMR	6(6bad)	1.3448×10^{-5}	0.0030
QC	18(3bad+15good)	9.7288×10^{-5}	30.2966

TABLE XII
COMPARISON BETWEEN MNMR ESTIMATOR AND QC ESTIMATOR FOR THE 4 BAD DATA CASE OF IEEE 118-BUS SYSTEM

Estimator	n_{bad}	E_V	E_θ
MNMR	4(4bad)	1.4202×10^{-9}	1.7200×10^{-4}
QC	5(4bad+1good)	7.9885×10^{-8}	2.2089×10^{-4}

The number of iterations and calculation time for the above test cases are shown in Table IX.

B. Comparison With the QC Estimator

In this section, we compare the two-stage MNMR estimator with QC estimator [14]. The parameters of the QC estimator are taken as the same value as [14]. The results are shown in Tables X–XII. $n_{\rm bad}$ is the number of bad data identified by estimators. The index E_V and E_θ defined by [14] are also listed. Their definitions are as follows:

$$E_V = \frac{1}{n}\sum_{i=1}^{n}(V_i - \hat{V}_i)^2$$
$$E_\theta = \frac{1}{n}\sum_{i=1}^{n}(\theta_i - \hat{\theta}_i)^2.$$

TABLE XIII
INFLUENCE OF λ IN THE FIRST STAGE ESTIMATION ON THE FIRST STAGE ESTIMATION RESULTS

λ	n_n	n_a	n_s	ratio
100	76(1bad+75good)	1(1bad)	39(4bad+35good)	0.0431
65	75(75good)	2(2bad)	39(4bad+35good)	0.0690
39.2948	80(80good)	3(3bad)	33(3bad+30good)	0.1034
35	83 (83good)	3(3bad)	30(3bad+27good)	0.1293
10	101(3bad+98good)	12(2bad+10good)	3(1bad+2good)	0.7931

Here n is the number of buses, V_i, θ_i are the actual bus voltage magnitude and angle at bus i and $\hat{V}_i, \hat{\theta}_i$ are the estimated values. (Voltages are measured per unit, angles in degrees.)

From Tables X–XII, we can see that the MNMR estimator can identify bad data correctly in all the three cases while the QC estimator fails. Especially in the IEEE 30-bus test case, the QC estimator only identifies 3 bad data and incorrectly rejects 15 good data. The voltage and angle performance indices for the MNMR estimator are usually better than or at least as good as the QC estimator. For the IEEE 30-bus test case, there is a distinct difference between the estimated angles and the true angles for the QC estimator and the angle index E_θ is greater than 30. However, the MNMR estimator can still estimate the angles precisely and the angle index E_θ is still very small.

Besides, in the IEEE 14-bus test case, the QC estimator wrongly identifies the correct measurement P_{5-1} as bad data because the bad data on P_{1-2} and P_1 are consistent; similarly in the IEEE 118-bus test case, the correct measurement P_{2-1} is identified as bad data due to the consistent property of the bad data on P_{1-2} and P_1.

C. Influence of Parameters on the Results

In this paper, the expanded uncertainty in measurement and the parameter λ in the second stage estimation are fixed. Therefore, in this section only the influence of λ in the first stage estimation will be discussed by taking the IEEE 30-bus with 6 bad data case as an example.

In Tables XIII–XIV, *ratio* is the same as that in Section A. The initial state in the first stage estimation is a flat start. The initial state in the second stage estimation is the solution of the first stage estimation.

n_n, n_a, and n_s are number of normal, abnormal, and suspicious measure points.

From Table XIII and XIV, we can see that when λ in the first stage estimation is too small, such as 10, *ratio* for the first stage estimation is too high so that the first stage estimation cannot obtain good solution with modern interior point method and thus the estimation results of the second stage estimation are also not satisfactory.

TABLE XIV
INFLUENCE OF λ IN THE FIRST STAGE ESTIMATION ON THE SECOND STAGE ESTIMATION RESULTS

λ	n_n	n_a	n_s	ratio
100	111(1bad+ 110good)	5(5bad)	0	0.2328
65	110 (110good)	6(6bad)	0	0.2586
39.2948	110 (110good)	6(6bad)	0	0.1897
35	110 (110good)	6(6bad)	0	0.1897
10	103(4bad+ 99good)	12(2bad+ 10good)	1(1good)	0.1034

TABLE XV
INFLUENCE OF λ IN THE FIRST STAGE ESTIMATION ON TIMING RESULTS FOR IEEE 30-BUS SYSTEM

λ	Iterations		Time	
	First stage	Second stage	First stage	Second stage
100	37	126	188ms	484ms
65	45	116	219ms	438ms
39.2948	34	52	188ms	203ms
35	70	54	313ms	203ms
10	278	9	1125ms	32ms

At the same time, when λ is too great, such as 100 in this case, the estimation results of the second stage estimation are not good, too. Although *ratio* in the first stage estimation is quite low, the too large λ makes the evaluation function of the first stage estimation too flat to lose too much of the bad data identification property.

Only when *ratio* for the first estimation is low and λ is not too great, such as 35, 39.2948, or 65 in this case, can the state estimation results of the two-stage estimation be good. Fortunately, the final estimation results are not sensitive to λ in the first stage estimation. For the IEEE 30-bus with 6 bad data case, the proposed estimation method can exactly reject bad data when λ in the first stage estimation ranges between 24 and 99. For values of λ in this interval, the estimation results are almost the same and the main difference lies in the calculation time.

The influence of λ in the first stage estimation on the timing of IEEE 30-bus case is shown in Table XV. It is obvious that too large λ and too small λ both lead to more iterations and longer calculation time.

VII. CONCLUSION

In this paper, a robust estimator is proposed based on the concept of normal measurement rate and the MCS estimator.

Similar to the MCS estimator, the MNMR estimator has distinct robustness, which has been successfully demonstrated through different IEEE AC examples and comparison with the QC estimator. This is because no exact fit of measurements is required and the MNMR estimator only seeks to find a solution with the highest normal measurement rate.

At the same time, the MNMR estimator overcomes the limitation of the MCS estimator to a large extent.

Firstly, the MNMR estimator has some noise filtering effect among the good measurements because we introduce the concept of suspicious measure point.

Secondly, the MNMR estimator formulates the state estimation as a nonconvex nonlinear optimization problem and solves this problem with modern interior point method, which results in high computational efficiency and allows the application to large-scale power systems.

The main problem of the MNMR estimator lies in its nonconvex property, which causes difficulty in finding the global optimum for the modern interior point method. In this paper, this problem is dealt with by using a two-stage estimation.

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Guangyu He (M'04) received the B.S. degree in automation and Ph.D. degree in electrical engineering from Tsinghua University, Beijing, China, in 1994 and 1999, respectively.

He is currently an Associate Professor of the Department of Electrical Engineering, Tsinghua University. His research interests are in the field of power system analysis and operations, and linear and nonlinear optimization for large-scale problems.

Shufeng Dong received the B.E. degree and Ph.D. degree in electrical engineering from Tsinghua University, Beijing, China, in 2004 and 2009, respectively.

He is currently doing postdoctoral research in the same department. His research interests include power system state estimation and optimization of the operation.

Junjian Qi received the B.E. degree in electrical engineering from Shandong University, Jinan, China, in 2008 and is currently a Ph.D. candidate of the Department of Electrical Engineering of Tsinghua University, Beijing, China.

His research interests include power system state estimation and its applications.

Yating Wang received the B.E. degree in electrical engineering from Tsinghua University, Beijing, China, in 2008, where she is currently pursuing the Ph.D. degree in the Department of Electrical Engineering.

Her research interests include distribution power system state estimation.